Solutions

Olympiad 1

Every 7 days from "today" will be Tuesday. Since 98 is a multiple of 7, the 98th day from today will be Tuesday. Then the 100th day from today will be Thursday.

Method 1

List the amounts in an organized manner.

	Amounts	Number
Amounts from 3¢-stamps:	3, 6, 9, 12	4
Amounts from 5¢-stamps:	5, 10, 15	3
Amounts from combining 3¢-stamps and 5¢-stamps:	3+5, 3+10, 3+15	3
	6+5, 6+10, 6+15	3
	9+5, 9+10, 9+15	3
	12+5, 12+10, 12+15	3
		Total 19

Method 2

The number of choices we have in using the 3¢-stamps is 5; we can use either 0, 1, 2, 3, or 4 of the 3¢-stamps. Similarly, we have 4 choices with respect to the 5¢-stamps; we can use either 0, 1, 2, or 3 of the 5¢-stamps. Each of the 5 choices for the 3¢-stamps can be combined with one of the four choices we have for the 5¢-stamps. This gives a total of 20 combinations. However, this total includes the combination of 0 3¢-stamps and 0 5¢-stamps. Since 1 or more of the stamps must be used, we exclude the combination of none of each. Therefore 19 different amounts of postage can be made.

Method 1

Given

(1) $S = 1 + 2 + 3 + \cdots + 23 + 24 + 25$

Reverse order of right side of (1)

(2) $S = 25 + 24 + 23 + \cdots + 3 + 2 + 1$

Add (1) and (2)

(3) $2S = 26 + 26 + 26 + \cdots + 26 + 26 + 26$

Simplify the right side of (3)

(4) $2S = 26 \times 25$

Divide both sides of (4) by 2

(5) $S = 13 \times 25$ or 325

The required sum is 325.

Method 2

Arrange the numbers in a square array as shown. Add the numbers in the left column (or bottom row). The sum of each of the other columns (or rows) can be easily determined by inspection. For example, each number in the second column is one more than its corresponding number in the first column. This is also true for other pairs of successive columns. The sum of the 5 columns (or rows) is 325.

21	22	23	24	25	115
16	17	18	19	20	90
11	12	13	14	15	65
6	7	8	9	10	40
1	2	3	4	5	15
55	60	65	70	75	325

Solutions

(Olympiad 1)

3) Method 3

The sum of the numbers in the third column (or in the third row) is 65. This is the average of the sums of the five columns (or rows). Multiply 65 by 5 to get the complete sum: $65 \times 5 = 325$.

Method 4

Examine the array of numbers shown in Method 2. Observe that the average of all numbers is 13, the number in the middle of the array. Then the sum must be $13 \times 25 = 325$.

4) Method 1

By combining both purchases we find that 5 pencils and 5 pens cost 150¢. Then 1 pencil and 1 pen cost 30¢, or 2 pencils and 2 pens cost 60¢. Since 3 pencils and 2 pens cost 72¢, 1 pencil costs 12¢.

Method 2

The difference in the prices of the two purchases is equivalent to the difference in the costs of a pen and a pencil. Therefore, a pen costs 6¢ more than a pencil, or, 3 pens cost 18¢ more than 3 pencils. Thus, the first purchase of 2 pencils and 3 pens is equivalent to the purchase of 2 pencils and 3 pencils plus 18¢, or 5 pencils plus 18¢. Since the cost of this purchase was 78¢, 5 pencils alone cost 60¢. Therefore 1 pencil had a cost of 12¢.

Method 3

Algebra: Let C represent the cost of one pencil and N the cost of one pen.

Given	(1)	2C	+	3N	=	78
Given	(2)	3C	+	2N	=	72
Multiply both sides of (2) by 3	(3)	9C	+	6N	=	216
Multiply both sides of (1) by 2	(4)	4C	+	6N	=	156
Subtract (4) from (3)	(5)	5C			=	60
Divide both sides of (5) by 5	(6)			C	=	12

Answer: A pencil costs 12¢

5) Each person of the work crew of three people worked 20 days. Thus the number of individual work days needed to do the job was 60. Then each member of the work crew of four people must work 15 days in order to provide a total of 60 individual work days.